as in Ref. 2, the new expression for Γ^* would be

$$\Gamma^* = 0.15/\chi^{1/3} \tag{14}$$

While this new value of Y_v substantially decreases the predicted vortex strength, Eq. (14) still overestimates the vortex strength by a factor of more than two. One further source of error may lie in the value chosen for Z_v. This has been taken from Fig. 5 of Ref. 3, which implies that the vortices have the same Z-coordinate as the midpoint of the jet plume. The vortices might actually lie below the line, which would require a smaller value of Z_v, and would further reduce the numerical constant in Eq. (14).

Conclusions

It appears that Thompson's data cannot be used to conclusively prove or disprove the validity of the author's model, because most of it lies outside the range of applicability of the model. His statement that a form of similarity variables which differs from that used by the author destroys the self-consistency of the model does not seem to be correct. As far as the vortex zone of the jet is concerned, Thompson's data sheds little light on the question of correlations for different values of σ_e due to the scarcity of points in this region, and to the possibility of wall effects influencing some of his data. Although some errors in the empirical constants of Ref. 2 have been corrected, the author's model still overpredicts the vortex strength by a factor of more than two. Further adjustment of empirical constants would require additional experimental information, such as vortex spacing and trajectory within the vortex zone of the jet plume.

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Comment on "A New Integral Calculation of Skin Friction on a Porous Plate"

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THE technique[1] of using a double-integral variation of the Kármán-Pohlhausen method was published by Whitehead[2] in 1949. The technique was applied to the case of the laminar boundary layer on an impervious surface and in a pressure gradient.

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Received July 14, 1971.
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Reply by Author to P. S. Granville

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THE author wishes to thank P. S. Granville for his interest and for bringing Whitehead's work[1] to the author's attention.

As Granville correctly pointed out, the idea of using double-integration had indeed appeared earlier in Ref. 1. However, it should be noted that Volkov's technique,[2] which was directly generalized in Ref. 3 to allow for surface mass transfer, was based on a somewhat different use of the idea. Particularly, the determination of skin friction in Ref. 2 differs from that in Ref. 1.

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Received October 8, 1971.
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Comment on "Large Deflection Analysis of Plates"

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IN his recent paper,[1] Yang in common with other workers[2] in the field of large deflections has used Levy's results[3] for a simply-supported square plate under uniform pressure as a standard against which the accuracy of his own finite element results may be assessed. In Ref. 4, a similar but less extensive study than Yang's of the nonlinear behavior of plates, employing conforming triangular finite elements,[5,6] revealed that Levy's original data was not sufficiently accurate for comparison with results from high-precision finite elements; moreover the boundary conditions employed by Levy were not identical to the usual kinematic constraints of a displacement finite element model.

The effect of employing more terms (n) in the Fourier Series expansion of Levy is shown below in Table 1

Table 1 Effect of additional terms on Levy's results

	Membrane $(\sigma/E)(1-\nu^2)\times10^4$	Bending $(\sigma/E)(1-\nu^2)\times10^4$	(w/t)
3	0.3865	0.3106	1.82444
5	0.3910	0.3624	1.84061
7	0.3901	0.3385	1.83681
9	0.3904	0.3505	1.83795
11	0.3903	0.3435	1.83750
13	0.3903	0.3478	1.83770
Levy[3]	0.392	0.384	1.846

Received August 30, 1971.
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extracted from Ref. 4 for a load parameter of $pa^4/Et^4 = 278.5$ and $\nu = 0.316$. Nondimensional forms of the central membrane and bending stresses (σ) and central deflection (w) are tabulated for a simply-supported square plate under a uniform pressure p, with side a, thickness t, Young's modulus E and Poisson's ratio ν.

It is evident that Levy's original data are in error, especially in the bending stress estimate.

The differences in boundary constraints are as follows; whereas the finite element in-plane boundary constraints stipulate and satisfy, at least for high-precision elements,[4] that the midplane displacements u and v are zero along a simply-supported edge, Levy[3] only constrains the average in-plane deflection normal to an edge to be zero. In addition, by choice of a midplane stress function, Levy forces the membrane shear stress along a simply-supported boundary to vanish, in contrast to the finite element constraints which permit a nonzero shear stress to exist along the edge. The influence of these constraint discrepancies may be small but could account for small differences between accurate finite element data and a refined and recomputed form of Levy's data.

A more suitable method for comparison purposes is that of Iyengar,[7] who employs trigonometric expansions for the in-plane displacements, rather than the midplane forces, and can thus exactly satisfy the condition of zero in-plane displacement normal to an edge, although the displacement along the edge is still left nonzero.

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Errata: "Swirling Nozzle Flow Equations from Crocco's Relation"

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[AIAA J. **9**, 1866–1868 (1971)]

EQUATION (14) of this Note can only be applied to a constant-area pipe because the radial velocity component was neglected in the energy Eq. (8) and the axial derivative of radial velocity was neglected in the Crocco Eq. (2). In this case, those arbitrary functions $C_1(z)$ and $C_2(z)$ appearing, respectively, in Eqs. (17), (18) and (23) are just constants. It should be noted that the radial velocity together with its derivatives should be retained for obtaining the axial flow variations along a nozzle.

Furthermore, the author made a statement "Mager employed a nondimensional velocity, $M = w/[a_0{}^2 - (\gamma - 1)w^2/2]^{1/2}$, which is not a Mach number as he claimed." The last three words "as he claimed" should be deleted. In a recent private communication with Mager, he indicated the M, which is called the Mach number of a "related" flow in his paper should be interpreted as, for example, one consisting of the axial velocity only but having the same total speed of sound as the swirling flow.

Received November 5, 1971.